

Minima and Maxima

4TH AUTUMN SERIES

DATE DUE: 6TH JANUARY 2014

Pozor, u této sérii přijímáme pouze řešení napsaná anglicky!

PROBLEM 1. (3 POINTS)

Ann found a triangle with numbers 1 to 6 written at its vertices and midpoints of its sides (each number was used once). When she summed the triplets of numbers along the triangle sides, the largest sum was 15. When she summed the pairs of numbers along the midlines, the smallest sum was 4. What is the largest possible sum of the three numbers at the vertices?

PROBLEM 2. (3 POINTS)

Numbers $1, 2, \dots, 2014$ are written around a circle in some order. What is the smallest possible sum of the absolute differences of adjacent numbers?

PROBLEM 3. (3 POINTS)

Martin loves every positive integer which does not contain digit 9 in its decimal representation and which becomes a square of an integer if any single of its digits is increased by one. Out of the numbers he loves, he loves the largest one the most. Which number is it?

PROBLEM 4. (5 POINTS)

In terms of n , what is the largest number of subsets of the set $\{1, 2, 3, \dots, n\}$ which can be chosen such that every two chosen subsets have at most two elements in common?

PROBLEM 5. (5 POINTS)

Given an equilateral triangle ABC , let ℓ be a line passing through A parallel to BC . For every point S on ℓ , consider a circle ω centered at S and tangent to the line BC . Determine all positions of S for which the length of arc of ω lying inside $\triangle ABC$ is the maximum possible.

PROBLEM 6. (5 POINTS)

Find the largest possible number of rooks that can be placed on a $3n \times 3n$ chessboard so that each rook is attacked⁴ by at most one other rook.

PROBLEM 7. (5 POINTS)

An uncolored 7×7 chessboard is given. What is the smallest number of squares which can be colored black so that every 5-square (Greek) cross contains at least one black square?

PROBLEM 8. (5 POINTS)

In a certain grocery store, Bartcha noticed 100 boxes full of fruits. Every box contained apples, bananas, and pineapples. Prove that Bartcha can buy 51 of these boxes so that she gets at least half of the apples, bananas and pineapples simultaneously.

⁴A rook is attacked by another rook if they belong to the same row or column and there are no other rooks between them.